

A casserole at temperature 40°F is taken from the refrigerator and placed on the table in a kitchen whose room temperature is 72°F. Simultaneously, an oven, set to 300°F, is turned on. Assume that the oven temperature, as it heats up, is described by the equation  $A(t) = 72 + 228(1 - e^{-\alpha t})^\circ\text{F}$ , where  $t$  is measured in minutes and  $\alpha$  is a positive constant. After 2 minutes, the casserole, resting on the table, has warmed up to 45°F and the oven temperature has risen to 150°F. At that time the casserole is placed in the oven. What is the temperature of the food 8 minutes after being placed in the oven?

The question in math terms:

$$A(0) = 72$$

$$A(2) = 150$$

$$A(t) = 72 + 228(1 - e^{-\alpha t}) = 300 - 228e^{-\alpha t}$$

$$T(0) = 40$$

$$T(2) = 45$$

$$\frac{dT}{dt} = k(A - T) \quad \text{find } T(10)$$

Physics note:

$k$  is the constant of proportionality. It tells how the object changes due to the environment it is in. Now, since the object is still the same object regardless of what environment, the  $k$  value stays the same whether the casserole is in the refrigerator, on the table, or in the oven.

Finding  $\alpha$ :

$$\begin{aligned} 150 &= A(2) = 72 + 228(1 - e^{-2\alpha}) \\ 78 &= 228(1 - e^{-2\alpha}) \\ \frac{13}{38} &= (1 - e^{-2\alpha}) \\ e^{-2\alpha} &= \frac{25}{38} \\ \alpha &= -\frac{1}{2} \ln\left(\frac{25}{38}\right) = \frac{1}{2} \ln\left(\frac{38}{25}\right) = 0.209355 \end{aligned}$$

Finding  $k$ :

When the casserole is on the table, the environment is constantly 72°F and we get:

$$\begin{aligned} \frac{dT}{dt} &= k(72 - T) \\ \int \frac{1}{72 - T} dT &= \int k dt \\ -\ln|72 - T| &= kt + C \\ |72 - T| &= e^{-kt+C} \\ T &= 72 - Be^{-kt} \end{aligned}$$

Now, we use our initial conditions:

$$\begin{aligned} 40 &= T(0) = 72 - B \\ B &= 32 \\ 45 &= T(2) = 72 - 32e^{-2k} \\ \frac{27}{32} &= e^{-2k} \\ k &= -\frac{1}{2} \ln\left(\frac{27}{32}\right) = \frac{1}{2} \ln\left(\frac{32}{27}\right) = 0.0849495 \end{aligned}$$

Finding T(t) when the Casserole is in the oven:

$$\frac{dT}{dt} = k(A(t) - T) = k[(300 - 228e^{-\alpha t}) - T] = 300k - 228ke^{-\alpha t} - kT$$

$$\frac{dT}{dt} + kT = 300k - 228ke^{-\alpha t}$$

$$\rho = e^{\int k dt} = e^{kt}$$

$$e^{kt} \frac{dT}{dt} + ke^{kt} T = 300ke^{kt} - 228ke^{kt} e^{-\alpha t}$$

$$\frac{d}{dx}(Te^{kt}) = 300ke^{kt} - 228ke^{(k-\alpha)t}$$

$$Te^{kt} = \int 300ke^{kt} - 228ke^{(k-\alpha)t} dt$$

$$Te^{kt} = 300e^{kt} - \frac{228k}{k-\alpha} e^{(k-\alpha)t} + C$$

$$T(t) = 300 - \frac{228k}{k-\alpha} e^{-\alpha t} + Ce^{-kt}$$

Now, we use our initial conditions and the  $k$  and  $\alpha$  that we found earlier.

$$45 = T(2) = 300 - \frac{228k}{k-\alpha} e^{-2\alpha} + Ce^{-2k}$$

$$45 = 300 - \frac{228(0.0849)}{(0.0849) - (0.209)} e^{-2(0.209)} + Ce^{-2(0.0849)}$$

$$C = -423.6165216$$

$$T(2) = 300 - \frac{228(0.0849)}{(0.0849) - (0.209)} e^{-10(0.209)} + (-423.6165)e^{-10(0.0849)}$$

$$T(10) = 138.036902388$$

Thus, we find that after 8 minutes in the oven (or 10 minutes after it was removed from the refrigerator) the casserole has warmed up to 138.0369°F.